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EXPERIMENTAL VERIFICATION OF AN APPROXIMATE TECHNIQUE FOR DAMAGE IDENTIFICATION IN BEAMS USING NONLINEAR REGRESSION ANALYSIS OF BENDING FREQUENCY CHANGES

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ABSTRACT:
This paper presents the experimental verification of an approximate technique for damage identification in beam-like structures, which is based on the use of bending frequency changes and nonlinear regression analysis. The proposed technique was explained in details in [7] so only few necessary steps are recalled here. The identification of damage location and its depth is performed by use of experimentally measured frequency changes and previously established regression relations between bending frequency changes and damage parameters. The experiments were performed on the intact and damaged beams. The damage in these experiments is made by saw cut as a narrow open notch perpendicular to the beam axis. The results of identification are promising taking into account inevitable modelling and measurement errors.
KEYWORDS:
damage identification, frequency chasnges, finite element analysis, nonlinear regression

1. INTRODUCTION

Damage in a structure changes its dynamic characteristics. Significant research has been conducted in damage detection and structural health monitoring using vibration-based techniques. The efforts in this area are still put towards developing new, more accurate and efficient damage detection techniques.

Modal parameters, such as natural frequencies, damping factors, and mode shapes are greatly affected by presence of a damage. These parameters can be measured more or less accurately using the available modal testing methods. An extensive survey of the literature dealing with methods for identifying cracks and damages was given by Dimarogonas [4] and Doebling et al. [5]. The use of frequency measurements in damage identification is very wide, mostly due to the fact that frequencies are relatively easy to measure. Salawu [15] presented damage identification techniques based on frequency information. Some of frequency-based techniques are presented in [1,10,11,12].

The aim of this paper is to present the experimental verification of the approximate damage identification technique that was presented in details in [7]. The technique is based on numerical data and regression relations between the bending frequency changes and damage parameters. Experimentally measured first four bending frequencies are used for identification of damage parameters (location and depth).

2. A BRIEF SURVEY ON THE IDENTIFICATION TECHNIQUE

A damage identification technique, which will be used here, is explained in details in [7]. A brief survey on the technique and the established regression relations will be recalled here.

A damage that occurs in a structure causes a change in mass and stiffness matrices and, consequently, in the natural frequencies. Using the matrix equation for undamped natural vibrations

$$M\ddot{q} + Kq = 0 \tag{1}$$

the eigenvalues i.e. undamped natural frequencies of the structure can be obtained. Here, the M represents the mass matrix, K is the stiffness matrix, and $\ddot{q}$ and q are the acceleration and displacement vectors, respectively.

On the basis of Eq(1), the eigenvalues of the intact and damaged structure can be obtained by numerical analysis. Although some analytical methods, for instance shown in [2,3,8,16], may also provide frequency values, which are the input data for regression analysis, the numerical method is much faster and easier.

The procedure proposed in [7] starts with the numerical model of the beam. Let the numerical model of the free-free beam has the following properties: length L_b=400 mm, height H=8,16 mm, width B=8.12 mm, modulus of elasticity E=2.068x10^{11} Pa, mass density ρ=7820 kg/m^3, and Poisson's coefficient ν=0.29, Fig.1. The damage is simulated as a narrow open notch perpendicular to the beam axis. The location of the damage is L_d , its depth is a, and the width of the notch is 1mm.

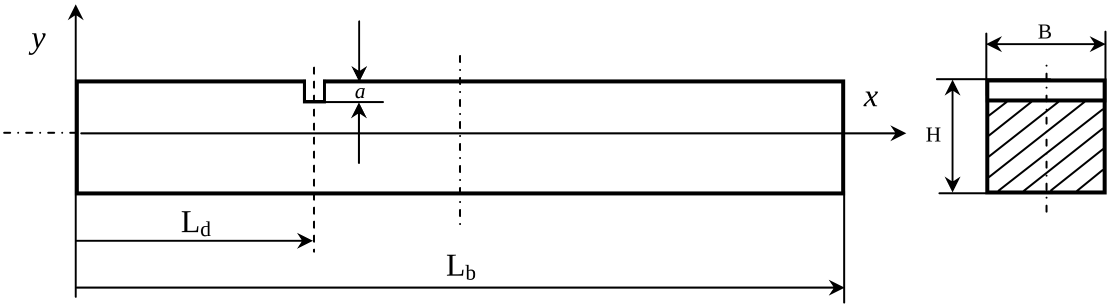


Figure 1. Geometry of the free-free beam with a notch

The beam is modeled using solid elements in software I-DEAS Master Modeler 9 and a number of numerical simulations were performed. Relative location L = L_d / L_b and relative depth of the damage D = a / H were varied with the aim to collect the values of the first four natural frequencies of the undamaged and damaged beams. Due to structural symmetry, the location of the notch L_d measured from the left end of the beam was varied from 10 mm to 200 mm in 10 mm increments. The depth a of the notch was varied from 1 mm to 4 mm in 1 mm increments.

Then, the relative frequency parameters $F_I = f_{I(d)} / f_{I(u)}$, I=1,2,3,4, were calculated. Here, f_I represents the Ith natural frequency, subscript (d) denotes the damaged beam, and subscript (u) refers to the undamaged beam structure.

Numerical values of the relative frequency parameters F_I, I=1,2,3,4, were taken as the input data for establishing the regression relations between the relative frequency changes and damage parameters (L and D). The Nonlinear Estimation option in the software STATISTICA 6.0 was used for statistical estimation of these regression relations, [17]. After several attempts, the best fit was chosen, taking into account the coefficient of correlation and shape of the nonlinear curve.

The following regression relations for the first four relative frequency parameters F_I, I=1,2,3,4, are obtained for the beam under consideration, [7]:

$$F_1 = 1 - 0.177566\ D^2 (-0.01948 - 0.85975\ L + 6.32585\ L^2 + 47.5372\ L^3 - 83.912\ L^4) \tag{2}$$

$$F_2 = 1 - 0.42922\ D^2 (0.065133 - 3.8651\ L + 45.7407\ L^2 - 44.275\ L^3 - - 267.41 L^4 + 406.144\ L^5) \tag{3}$$

$$\begin{aligned} F_3 = 1 - 11.0353\ D^2 (0.006469 - 0.37663\ L + 5.74127\ L^2 \\ - 16.533\ L^3 - 32.81 L^4 + 180.968\ L^5 - 177.36\ L^6) \end{aligned} \tag{4}$$

$$\begin{aligned} F_4 = 1 - 69.938\ D^2 (0.00252 - 0.17049\ L + 3.30183\ L^2\ 18.862\ L^3 \\ + 21.3852\ L^4 + 121.863\ L^5 - 375.07\ L^6 + 298.339\ L^7) \end{aligned} \tag{5}$$

As can be seen, these regression relations include the quadratic influence of the relative depth D and polynomial influence of the relative location L. The corresponding coefficients of correlation are very high (0.996 for F_1, F_2, F_3, and 0.998 for F_4). The obtained regression curves along with the numerically obtained relative frequency changes for different relative locations and some relative depths are presented in Fig.2.

It can be seen that the largest differences between the values obtained numerically and those calculated by regression relations given by Eqs.(2)-(5) are at those locations of the beam where modal nodes and modal extreme points occur. This results from the nature of regression relations to smooth the data at extreme points.

The obtained regression surfaces, if cut by an appropriate horizontal plane that corresponds to a specific value of the frequency change, gives the intersection curves in D-L plane. These intersection curves contain all possible values of D and L that correspond to the same frequency change, Fig.3.

The intersection curves should theoretically give a common intersection point (D_{est}, L_{est}) in D-L plane, which shows the estimation of damage parameters, see [13,14,18]. However, in practice, it would be very hard to obtain the same intersection point for all four frequency curves as the numerical values of intersection point coordinates can differ more or less.

It is also obvious that changes in only two first frequencies should be sufficient to find the characteristic intersection point. However, due to inevitable modeling errors (i.e. differences between the numerical model and the real structure) and measurement errors, one should not rely solely on only two measured frequency changes to find the intersection point (D_{est}, L_{est}). It is much safer and reliable to locate this point using more frequency changes, which also contribute to the robustness of the technique. Basically, the first four frequencies are shown to be sufficient to identify a crack in the beam, [18].

Also, as shown in Figure 3, there are another intersection points in the D-L plane between the curves obtained by a pair of frequency changes. However, these intersection points are isolated and not confirmed by the other frequency changes so in analytical approach (without graphical presentation) they should be eliminated.

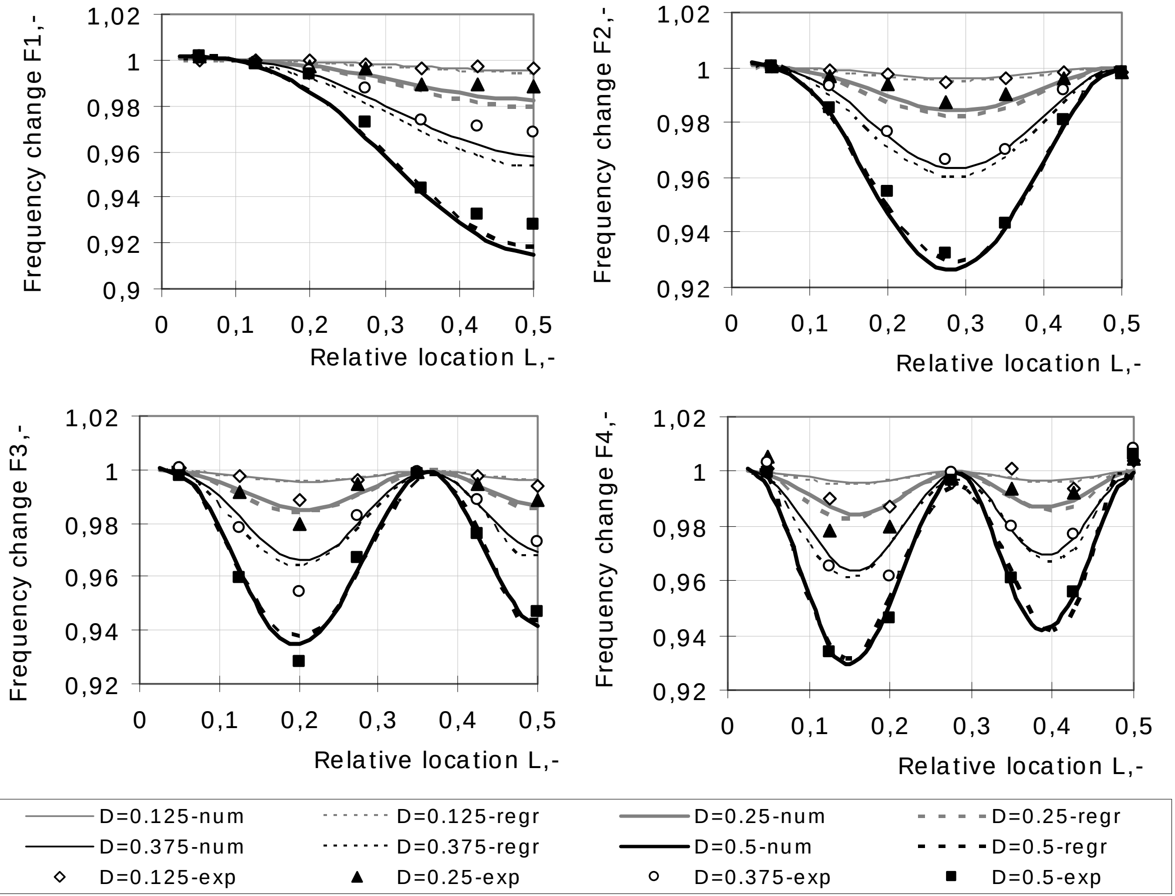


Figure 2. Relative frequency changes F_1- F_4 in relation to the relative location l and relative depth d of the notch obtained by numerical model, regression expressions and experimentally

Using an appropriate mathematical method to solve the system of regression relations, one can found an appropriate set of intersection points. Next, the technique proposed in [7] suggests finding the three closest intersection points in the appropriate set of intersection points, i.e. those that give minimal sum of their distances from their mean value. The coordinates of their mean value can be adopted to represent the damage parameters. The only prerequisite here is that this mean value should not estimate the damage much beyond the numerically observed ranges of L and D, i.e. the ranges that are covered by the adopted regression curves (here, the range is: L=0.025 to 0.5, D=0.125 to 0.5). In an opposite case, the next combination of three intersection points giving minimal sum of distances from their mean value should be appropriate.

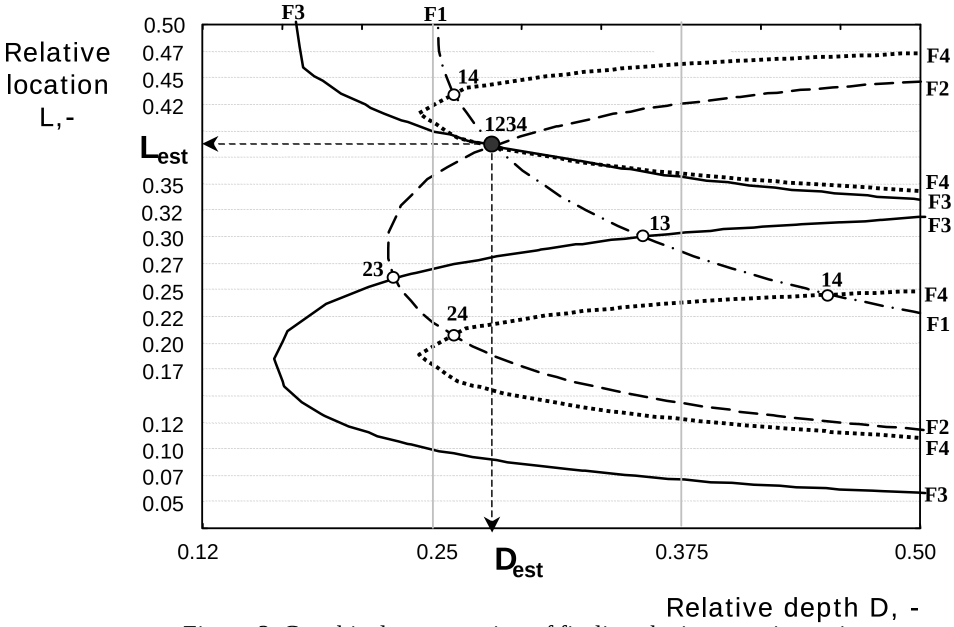


Figure 3. Graphical presentation of finding the intersection point
Of frequency curves in D-L plane (theoretical case)

3. ACQUISITION OF THE EXPERIMENTAL DATA

The proposed procedure will be checked by experimentally measured values of frequency changes, [6].

The experimental setup is shown on Fig. 4a. The instruments used in this experimental study were: impact hammer B&K 8202 with load cell B&K 8200, accelerometer B&K 4394, frequency analyzer HP 3567A, interface HP82335A and a personal computer.

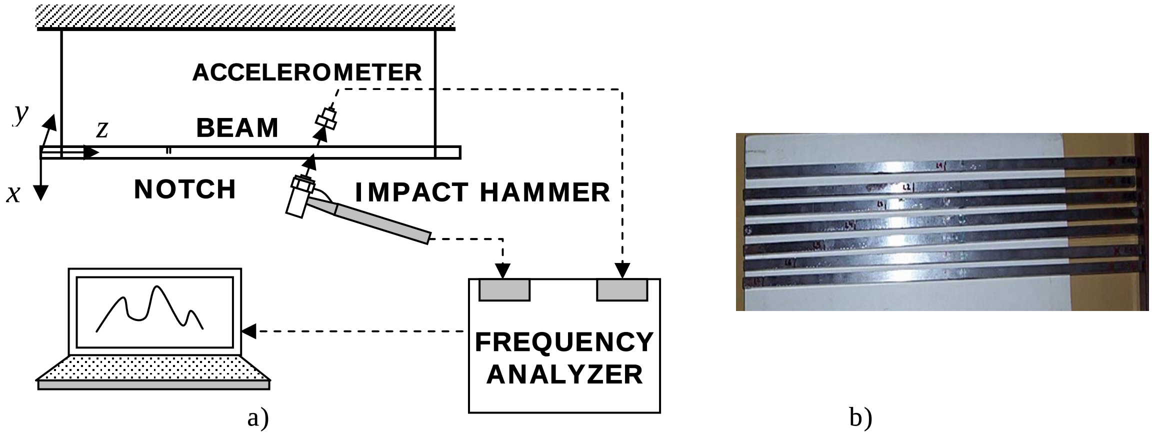


Figure 4. Experimental setup: a) scheme; b) seven beam samples

Experiments were performed on seven beam samples in free-free boundary conditions, Fig.4b. The free-free state is attained by hanging a beam using two thin silicon ropes. Cracks of varying depths are made with a saw at different locations on beams. Since the crack is made with a thin blade (1mm in width), the crack always remains open during the vibration test. Each beam model was made of a steel bar with a nominal cross-sectional area 8 mm by 8 mm and a length of 400 mm. The frequency measurement was performed twice for each of 7 undamaged beams and 28 combinations of crack locations and depth.

The previously established numerical model and regression curves were used for all measured beams despite slight discrepancies in their dimensions and frequency values in undeformed state, Fig.5. This means also that there were the differences between the unique numerical model and these seven measured beams, but on the other hand this enabled to check the robustness of the proposed technique. Of course, in real practice, the efforts should be put to establish the numerical model that fits the real structure the best, which will give more chance to the better identification results.

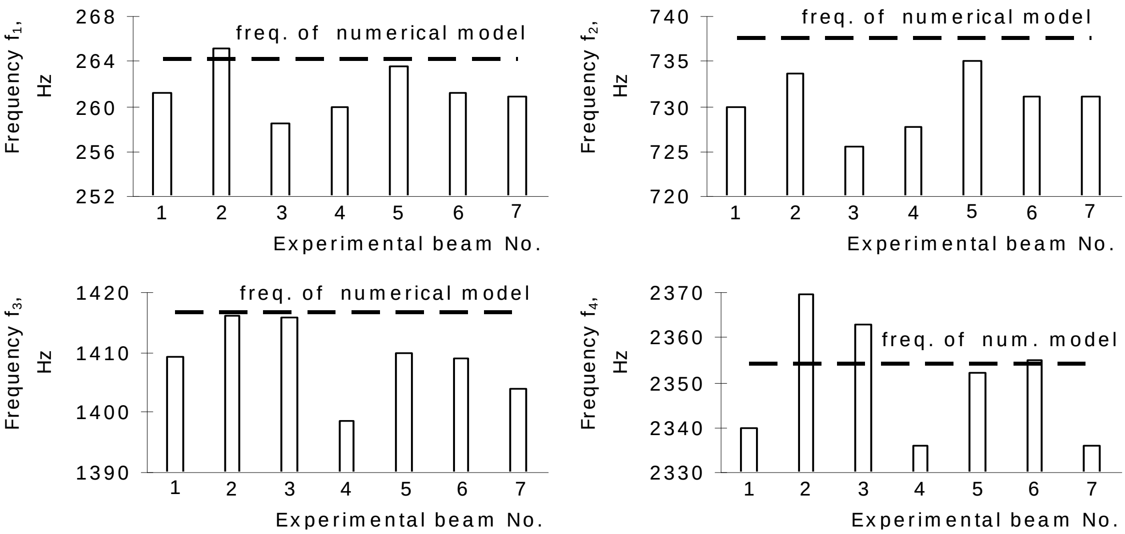
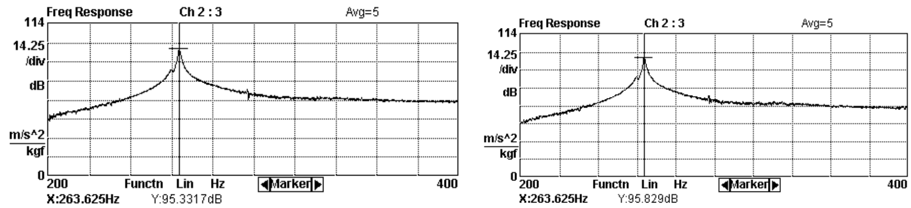
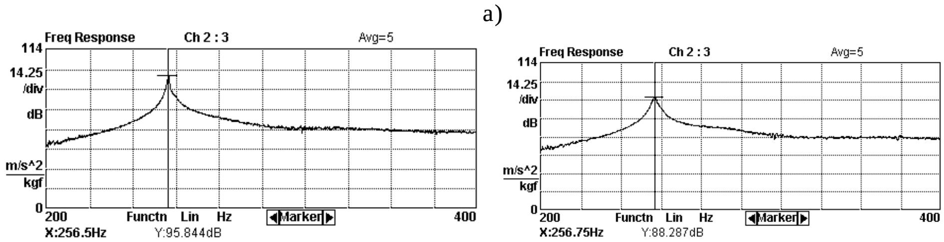


Figure 5. The first four bending frequencies of the measured undamaged beams in comparison to the frequencies of the numerical model



a)



b)

Figure 6. Two independent measurements of the first frequency of the beam sample: a) undamaged beam, b) damaged beam - case: L=0.35, D=0.375

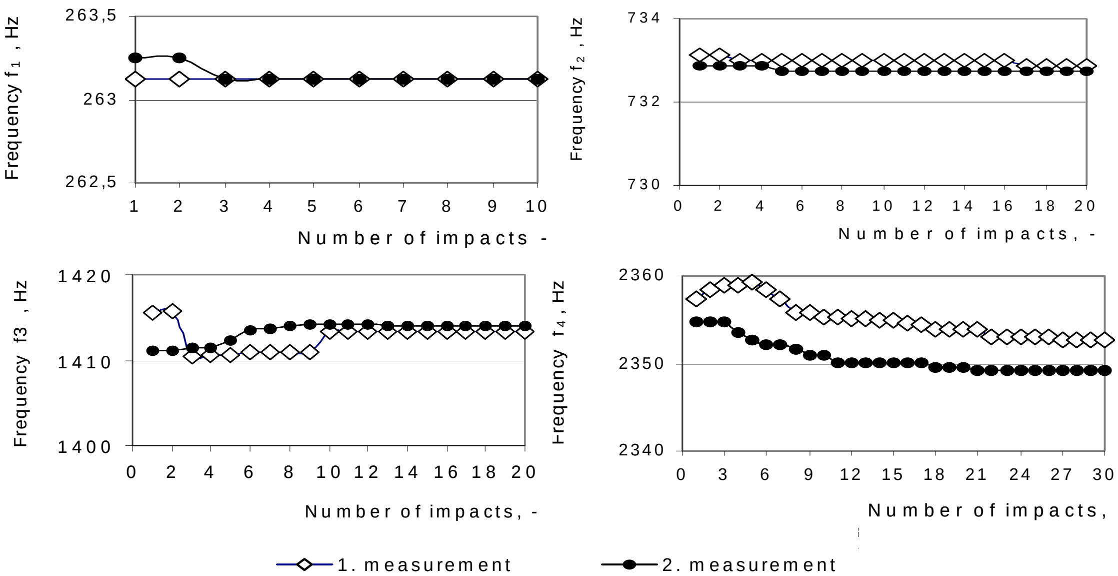


Figure 7. The values of the first four bending frequencies in dependence on the number of impacts for two independent measurements

The mean value of frequencies obtained by two independent measurements of undamaged and damaged beams, Fig.6, was used to calculate the frequency ratios F_I, I=1,2,3,4.

It should be stated here that the value of the measured frequency depends on the frequency and resolution range, and the number of impacts. These ranges were adopted during measurements:
- 1th frequency: frequency range 200-400 Hz, resolution 0.125 Hz, 5 impacts,
- 2nd & 3rd frequency: freq. range 600-1600 Hz, resolution 0.25 Hz, 10 impacts,
- 4th frequency: frequency range 1.5-5.5 kHz, resolution 1 Hz, 10 impacts.

The number of impacts were chosen in dependence of the beginning of frequency value stabilization, Fig.7, except for the fourth frequency where higher number of impacts (20 or 30) would require a lot of measurement time.

The results of experimental measurement of natural frequency changes are shown in Fig. 7. It is obvious that these experimental values differ more or less from those calculated for the unique numerical model of the beam that is used to represent all of the seven measured beams. Besides the modelling discrepancies, this is partially due to less accurate measurement of low frequency changes (inherent to small relative depths D) and inability to accuratelly cut the nominal depth of the notches.

4. DAMAGE IDENTIFICATION USING EXPERIMENTAL RESULTS

An example of identification. Let's reveal the damage parameters D and L in case that the measured frequency changes are the following: F_1= 0,973428, F_2= 0,970231, F_3= 0,999468, F_4= 0,980017.

Using the regression expressions (2)-(5) and all pairs of two measured frequencies, the real values of L and positive values of D can be calculated. These pairs of D and L make an appropriate set of intersection points, Table 1.

Table 1. The values of relative location L and relative depth D calculated for all combinations of the first four frequencies (for experimental example)

Frequency curves	Point No.	Relative depth D	Relative location L	Frequency curves	Point No.	Relative depth D	Relative location L
F_1& F_2			-0,44358	F_1& F_4			-0,35632
F_1& F_2			0,038303	F_1& F_4			0,028291
F_1& F_2			0,054851	F_1& F_4			0,04493
F_1& F_2	No.1	0,353376	0,344738	F_1& F_4	No.6	0,53491	0,240988
F_1& F_2	No.2	0,315118	0,568344	F_1& F_4	No.7	0,33754	0,362171
				F_1& F_4	No.8	0,303154	0,416302
				F_1& F_4	No.9	0,289114	0,520827
F_1 & F_3			-0,33236	F_2 & F_4	No.10	0,16204	-0,35515
F_1 & F_3			0,028603	F_2 & F_4			0,026761
F_1 & F_3			0,053006	F_2 & F_4			0,044545
F_1 & F_3	No.3	0,293947	0,535885	F_2 & F_4	No.11	0,367256	0,212232
F_1 & F_3				F_2 & F_4	No.12	0,363009	0,353158
F_1 & F_3				F_2 & F_4	No.13	2,579492	0,476197
				F_2 & F_4	No.14	2,483278	0,499449
F_2 & F_3	No.4	0,149056	-0,33211	F_3 & F_4			-3,29249
F_2 & F_3			0,028539	F_3 & F_4	No.15	0,032081	-0,32765
F_2 & F_3			0,052999	F_3 & F_4			0,028619
F_2 & F_3	No.5	0,567081	0,536263	F_3 & F_4	No.16	1,249093	0,053548
F_2 & F_3				F_3 & F_4	No.17	0,178665	0,534983
F_2 & F_3				F_3 & F_4			
				F_3 & F_4			

Note: Empty spaces in the column D refer to complex values

The procedure of finding the closest three points between these 17 points gave the following results: D_{est}= 0,351308 and L_{est} =0,353356 for the relative depth and relative location. In absolute values, the estimated location of the damage is 141,3423 mm measured from the left end of the beam and the value of damage depth is 2,810467 mm (the actual location is 140 mm and the nominal depth is 3 mm). This estimation is made by intersection points No. 1, 7 and 12, i.e. by F_1, F_2 and F_4. The third frequency change has a nodal point close to this damage location, so it was unable to accurately locate the damage.

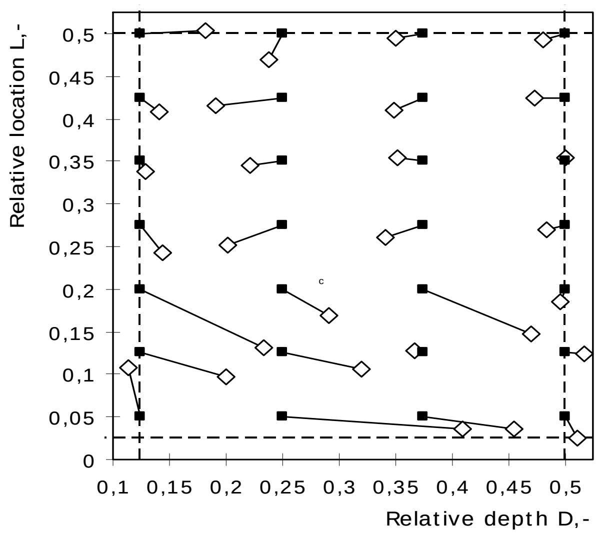


Figure 8. Results of damage identification using experimentally measured frequency changes (■ Actual Damage Parameters, ◇ Estimated Damage Parameters)

The identification procedure described above was repeated for 28 cases of damage parameters (7 damage locations with 4 depths) using the measured frequency changes.

The results of identification are shown in Fig. 8. It is obvious that despite of all the previously mentioned unadequacies and limitations, the results of the adopted identification technique in real conditions are quite satisfactory. If the allowed error for the absolute location is ±20mm, then only 3 of 28 locations are estimated incorrectly, giving rise to the identification accuracy of location of 89.28 %. If the appropriate error for the absolute depth is ±0.5mm, then 5 of 28 depths are incorrectly identified, so the identification accuracy of the depth is 82.14 %.

The identification procedure gave less accurate results in cases of small frequency changes due to difficulties to accurately measure them. Also, the identification of damages located at modal nodes and modal extreme points are less accurate due to the smoothing property of regression relations. As expected, the best results are obtained for damages with larger relative depths. For better results i.e. for a specific real beam the efforts in establishing good numerical model should be previously put.

4. CONCLUSION

The results of experimental verification of the approximate identification technique introduced in [7] are presented in the paper. For experimentally measured frequency changes, the results of damage identification are quite satisfactory expecially for greater depth of the damage. The identification of minor damages could be improved with numerical model updating, for instance in [9], better mesh refinement with consequent higher number of numerical data required for establishing the regression relations. However, the measurement of small frequency changes due to minor damages represents an inherent problem in real practice.

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